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RESEARCH ARTICLE

Attribute Based Analysis, Evaluation and Selection of Nanocomposite Using MADM-TOPSIS Approach

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Abstract

The use of nanocomposites is rapidly growing in various fields for different applications like construction, electronic, automobiles etc. Nanocomposites possess unique size dependent material properties. The various types of nanocomposites with different sizes and shapes such as particulate, fibrous, layered are good alternatives for conventional composites. The problem of selection of nanocomposite materials has been a concern to users for recent years. This problem has become more difficult due to complexity of nanocomposites. The objective of this study is to maintain reliable database based on their different pertinent attributes. This database can be used for nanocomposite selection procedure when user has decided to use the nanocomposite material for some purpose and this will help the user to save time. The selection procedure proceeds to rank the alternatives in the shortlist by employing different attributes based specification methods and graphical methods. This ranking will provide a good guidance for users to select a material. A three stage selection procedure is used for identification of pertinent attributes and ranking is done with MADM-TOPSIS approach.

Keywords: Nanocomposites, reliable database, pertinent attributes, conventional composites, MADM-TOPSIS.

Introduction

Composites are considered to be combination of materials differing in composition or form on a macro-scale. The constituents maintain their identities in the composite, i.e. they do not get dissolve or otherwise amalgamate completely into each other although they act in concert. Nanocomposites are materials in which one of the components has at least one dimension that is a nanoscopic in size that is 10^{-9} m. The materials used for nanocomposites are like ash and clay. Depending on the nature of the nano-phase and the matrix, a wide range of nanocomposites can be prepared. These composite materials can assume a mixture of the beneficial properties of their base compounds, leading to materials with improved physical properties and unprecedented flexibility among them. A Multi-Attribute Decision Making (MADM) –Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a methodology for evaluation, coding, ranking and optimum selection of sub-systems for composite product used directly by its manufacturers. This method is important from the point view of development of a reliable database, virtual design and customization, developing cutting-edge technology and meeting the challenges of global competition in composite industry (Prabhakarn *et al.* 2006). TOPSIS based evaluation and ranking, other graphical methods (linear graph and spider diagram), works on the information of the pertinent attributes. This procedure ranks the mechatronic system alternatives based on the Euclidean distance of alternatives from hypothetically best and hypothetically worst mechatronic systems (Kiran *et al.,* 2011). The objective of this study is to maintain reliable database based on their different pertinent attributes. This database can be used for nanocomposite selection procedure when user has decided to use the nanocomposite material for some purpose. Therefore, a methodology is required by which design, evaluation and selection of nanocomposite can be made comprehensive and easy, considering various factors affecting the selection. In order to accomplish the above objective, the system approach named MADM-TOPIS approach is suggested, which consist of:

i. Identification of attributes of Nanocomposites.
ii. Development of N-digit coding scheme to collect the information about the each attribute.
iii. Development of TOPSIS procedure for attribute based evaluation, comparison, design improvement and ranking of feasible alternatives.
iv. Final stage selection considers all aspects which have not been considered during evaluation and ranking procedure.

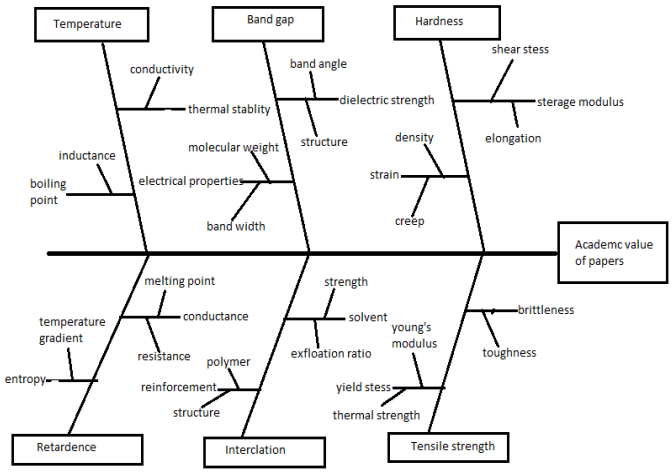
Materials and methods

Methodology: TOPSIS method is a popular approach to MADM used widely. This approach was first developed by Hwang and Yoon (1981) for solving a MADM problem. In this methodology, we simultaneously considers the distances to the positive bench mark and negative bench mark regarding each alternative and selects that which is

close to the ideal solution as the best alternative. The best alternative is the nearest one to the ideal solution and the farthest one from the negative ideal solution (Chanodrakas *et al.*, 2011). The advantage, the approach has is the ability to identify the best solution quickly. As the first process, a list of all the attributes were prepared and then make the cause and effect diagram to identify them (Prabhakarn *et al.*, 2006). Then, every attribute is coded in the interval scale of 0-5 based on the importance of the attribute. Then, the elimination search is done to evaluate the optimum or best solution.

Nanocomposite attributes: Cause and effect analysis is used for identification of different attributes and their effect to decide the academic value of different attributes. Proper identification of nanocomposite attributes is critically important when comparing various alternative nanocomposites. However, in most cases, the user needs to be assisted in identifying the nanocomposite attributes wisely and accordingly the application. The final product of industry directly depends on the proper choice of the nanocomposite. Cause and effect analysis diagram is drawn to identify all the different attributes and other parameters of nanocomposite which require attention/situation of researchers in the subject area under consideration as shown in Fig. 1.

Fig. 2. Cause and effect analysis.



Identification of attribute: In global market, nanocomposites are provided by manufacturers with different type of properties for different purposes. User's ultimate objective is to acquire and utilize a best nanocomposite that will provide the most appropriate performance on a whole of life basis, in terms of capital, replacement and maintenance costs. All the users have different functional requirements and usage criteria. Therefore, the optimum selection of nanocomposite product system for a particular application is going to affect the overall performance of the product and at the same time the selection of nanocomposite is mainly dependent on its different attributes, e.g. tensile strength, tensile modulus and elongation break, etc. For evaluating and comparing the various alternative nanocomposites, the proper identification of the system attributes is very much important. Selecting the best nanocomposite for a particular application must be carefully considered and researched by the consultant or engineer in close coordination with the designer. It is difficult for manufacturer to select a nanocomposite for a given application. The designer has to keep in mind the quality of product and the satisfactions of the customer. So keeping all this in mind, nanocomposite attributes based on its broad area as general parameters, design aspects, nanocomposite characteristics, etc. are identified. They are listed in Table 1. Here, 153 pertinent attributes necessary to design and manufacture the nanocomposites or nanocomposite products are identified, but the user, designer or manufacturer can add or delete some of the attribute depending upon their requirement. If the identification of nanocomposite attributes is done carefully, the selection nanocomposite for typical application will be precise, which will satisfy the user's requirements.

N–Digit coding scheme for attributes: The next step is to assign codes to the attributes which is either a numerical value or an alphabet. This is done under the coding scheme which is very important as it gives all the detailed information about the attributes. There is qualitative and quantitative type of attributes in nature. The qualitative attributes are of subjective or fuzzy type whereas the quantitative attributes are deterministic. The quantitative attributes are given codes on an interval scale of 0-5, where 0 indicates that no information is available about the attribute and 5 indicates the best information and alternative present in the attribute.

Proposed coding scheme is illustrated in Table 4 for a nylon 6/clay nanocomposite. The first column represents the block number corresponds to the 153 attributes, the second column represents the name of the attribute, the third column represents information about the attribute in a particular application and the fourth column represents the alphanumeric code based on the type of attribute. The classification of the attributes is done either quantitatively or qualitatively. A compact coding structure has been prepared for the above application and is shown in Table 3. In this coding structure, the code within the box is in format y_i/x_i, i=1… n, where y_i is the block number and x_i represents the alphanumeric code allotted to the particular attribute. For qualitative, the coding is done on a grade scale. Some of the attributes are coded with alphabets. A numerical value is allotted to quantitative attribute. Numerical values of the quantitative attributes are allotted in scaled ranges 0-3 and 0-5. The proposed coding scheme is illustrated here with examples. First example is for quantitative attributes is given in Table 2.

Table 1. Different categories of nanocomposite attributes.

General attributes	
1. Clay	2. Organoclay
3. Polymer	4. Filler
5. Transparency	6. Fiber
7. Silica	8. Glass
9. Inorganic	10. Transparent
11. Organic	12. Color
13. Viscoelascity	14. Rheological properties
15. Epoxy	16. Angle
17. Resin	18. Blends
19. Oxygen	20. Nitrogen
21. Biodegradable	22. Corn starch
23. Elasticity	24. Exfloation Ratio
25. Isostatic	26. Carbon
27. Laminated	28. Reinforcement
29. Elastomer	30. Tribological properties
31. Aspect ratio	32. Rubber
33. Polyester	34. Flexibility
35. Ultrafine	36. Extrusion
37. Biochemical	38. Nanotube
39. Micro	40. Cluster
41. Conventional	42. Nano metric range
43. Water content	44. Softness
45. Durability	46. Reflection
47. Nanofiber	48. Carbon nanotube
49. Nano Silica	50. Nano titanium oxide
51. Nylon	52. Polyolefin
53. Polyamides	54. Polyurethanes
Chemical attributes	
55. Silicate	56. Situ polymerization
57. Hydrolysis	58. Condensation
59. Terephthalic acid	60. Ethylene Glycol
61. Crystallization	62. Solvent
63. Tolerene	64. Xylene
65. Chloroform	66. Evaporation
67. Dispersion	68. Intercalation
69. Equilibrium	70. Crystallization
71. Ion exchange	72. Methacrylate
73. Alkylammonium	74. Phosphonium
75. Extrusion	76. Dilution
77. Montmorillonite	78. Reactivity
79. Adsorption	80. Nucleation
81. Graphene	82. Citric acid
83. Fomamide	84. Concentration
85. Polyvinylprolidone	86.Oxidation
87. Toxicity	88. Band Gap
89. Chemical properties	
Mechanical attributes	
90. Shear stress	91. Tensile strength
92. Young's modulus	93. Strain
94. Creep	95. Bending strength
96. Structure	97. Elongation break
98. Fracture	99. Toughness
100. Ductility	101. Buckling stability
102. Stiffness	103. Pressure
104. Tensile modulus	105. Permeability
106. Tensile stress	107. Deformation
108. Storage modulus	109. Brittleness
110. Impact strength	111. Hardness
112. Mechanical properties	113. Malleability
114. Flexural strength	
Thermal attributes	
115. Melting point	116. Boiling point
117. Heat	118. Thermal Stability
119. Thermal properties	120. Temperature
121. Thermo gravimetric	122. Thermoplastic
123. Heat resistance	124. Flammability
125. Entropy	126. Heat source
127. Combustion	128. Isothermal
Electrical attributes	
129. Dielectric	130. Conductivity
131. Inductance	132. Threshold
133. Capacitance	134. Electrical properties
Physical attributes	
135. Molecular weight	136. Surface area
137. Cylindrical shells	138. Diameter
139. Phase	140. Moisture
141. Density	142. Purity
143. Surface tension	144. Gas
145. Vaporization	146. Length
147. Purity	
Organic attributes	
148. Plasticizer	149. Poly carbonate
150. Phosphonium	151. organomodifield
152. Mass ratio	153. organoclay

Table 2. Coding of storage modulus of nanocomposites.

Storage modulus in Gpa	Code
Up to 0.19	0
0.43	1
1.97	2
2.08	3
3.71	4

Quantitative attributes: This code will be used to specify the importance of storage modulus in the respective block number 108, since it is allotted to it as shown in Table 1. These codes are used to specify the storage modulus of the nanocomposite. Here, the nanocomposite under consideration has the specific storage modulus of 4.14 Gpa, which is given a code 5.

Qualitative attribute: Suppose, we are coding the bending strength of nanocomposite, it can be done as shown in Table 3.

Table 3. Coding of storage modulus of nanocomposites.

Storage modulus in Gpa	Code
Up to 80	0
100	1
110	2
122	3
135	4
150	5

Coding scheme for standard attributes of Nylon6/Clay nanocomposite: In Table 4, the code 0, represents that the information relating to the particular cell is not available to the authors. This information is not provided by the manufacturer, but the authors feel that this information should be provided by the manufacturer to make the database exhaustive. Moreover, the database storage, retrieval and the selection procedure will be more precise and accurate. This coding scheme can be used for comparing two nanocomposites critically and for better insight and understanding of a nanocomposite alternative or the subsystem or the components from the database. The alphabets used in the coding scheme for standard attributes has unique information in itself like, "T" represents the transparent property of nanocomposites, F represents the fiber, "G" represents the Glass material, Black colour represented by "C", elasticity represented by "E".

Table 4. Coding scheme for Nylon6/Clay nanocomposite.

S.No	Content	Information	Code
1	Clay	Clay content up to 5 wt%	5
2.	Organoclay		2
3.	Polymer	Polymer used	4
4	Filler	Filling material	5
5.	Transparency		0
6.	Fiber		F
7.	Silica	Content 4	4
8.	Glass	G	G
9.	Inorganic		0
10.	Transparent property	Property	T
11.	Organic		0
12	Color	Black	C
13	Viscoelascity		0
14.	Rheological properties		0
15.	Epoxy	Material	5
16.	Angle		0
17.	Resin		2
18.	Blends		0
19.	Oxygen		0
20.	Nitrogen		0
21.	Biodegradable		0
22.	Corn starch		0
23.	Elasticity		E
24.	Exfloation ratio	Exfloated up to 20 wt%	4
25.	Isostatic	Phase	2
26.	Carbon		0
27.	Laminated		0
28.	Reinforcement		0
29.	Elastomer		0
30	Tribological Properties		TP
31	Aspect ratio	> 1000	A
32.	Rubber	Material	R
33.	Polyester		0
34.	Flexibility		0
35.	Ultrafine		0
36.	Extrusion	Process	ER
37.	Biochemical		0
38.	Nanotube		0
39.	Micro		0
40.	Cluster		0
41.	Conventional		0
42.	Nano metric	Range 10^{-9} m	5
43.	Water content		0
44.	Softness		0
45.	Durability		0
46.	Reflection		0
47.	Nanofiber		0
48.	Carbon nanotube		0
49	Nano silica		0
51.	Nylon		0
52.	Polyolefin		0
53.	Polyamides		0
54.	Polyurethanes		0
55.	Silicate		0
56.	Situ polymerization	Process	SP
57.	Hydrolysis		0
58.	Condensation		0
59.	Terephthalic acid		0
60.	Ethylene glycol		0
61.	Crystallization		0
62	Solvent	Material	S
63.	Tolerene		0
64.	Xylene		0
65.	Chloroform		0
66.	Evaporation		0
67.	Dispersion		0
68.	Interclation	Interlayer spacing 4.0 nm	I
69.	Equilibrium		0
70.	Crystallization		0
71.	Ion exchange		0
72.	Methacrylate		0
73.	Alkylammonium		0
74.	Phosphonium		0
75.	Extrusion		0
76.	Dilution		0
77.	Montmorillonite		0
78.	Reactivity		0
79	Adsorption		0
80.	Nucleation		0
81.	Graphene		0
82.	Citric acid		0
83.	Fomamide		0
85.	Polyvinylprolidone		0
86.	Oxidation		0
87.	Toxicity		0
88.	Band gap		0
89.	Chemical properties		0
90.	Shear stress		0
91.	Tensile strength	Up to 97 Gpa	5
92.	Young's modulus	Up to 6 Gpa	5
93.	Strain		0
94.	Creep		0
95.	Bending strength		4
96.	Structure		0
97.	Elongation		0
98.	Fracture		0
99.	Toughness		3
100.	Ductility		2
101.	Buckling Stability		3
102.	Stiffness		0
103	Pressure	Force per unit area	P
104.	Tensile modulus		5
105.	Permeability		0
106.	Tensile stress		0
107.	Deformation		0
108.	Storage modulus	Up to 4.41 Gpa 5	5
109.	Brittleness		0
110.	Impact		0
111.	Hardness		0
112.	Mechanical properties		0
114.	Flexural strength	Up to 40 Mpa	4
115.	Melting point	3700 to 3800°C	5
116.	Boiling point		0
117.	Heat		0
118.	Thermal stability	Retardancy to heat	TS
119.	Thermal properties		0
120.	Temperature		3
121	Thermo gravimetric		0
122.	Thermoplastic		0
123.	Heat resistance		0
124.	Flammability		0
125.	Entropy		0
126.	Heat source		0
127.	Combustion		0
128.	Isothermal		0
129.	Dielectric		0
130.	Conductivity		0
131.	Inductance		0
132.	Threshold		0
133.	Capacitance		0
134	Electrical properties	Conducting	EP
135	Molecular weight		4
136	Surface area		5
137.	Cylindrical shells		0
138.	Diameter	4 nm to 12nm	4
139.	phase	phase	0
140	moisture	Absorbable	3

141.	Density		0
142.	Purity	> 97% by weight	5
143.	Surface tension		0
144.	Gas	State	G
145.	Vaporization		0
146.	Length	2 to 6 mm	5
147.	Purity		0
148	Plasticizer		0
149.	Poly carbonate		0
150.	Phosphonium		0
151.	organo-modifield		0
152	Mass ratio		0
153.	Organoclay		0

Tribological properties represented by "TP", rubber by "R", extrusion by "ER", situ polymerization by "SP", "S" Represents the solvent, intercalation by "I", pressure represented by "P", "TS" represents the thermal stability, electrical properties represented by "EP" and gas represented by "G". Rest of numerical codes are given on the basis of their relative importance highest code i.e. "5" to highly important attribute and lesser code like "4", and "3" to less important and "2", and "1" to very less important attribute and "0" for totally absent attribute. This coding scheme is also used for the visual comparison between two nanocomposites up to certain extent. It allows faster comparison in various formats.

Optimum selection procedure: There are two main stages used in the optimum selection procedure as follows:
Stage-1-Elimination search method: Though all the attributes are identified, all of them are not essential to be considered while selecting the nanocomposite for particular application, only few attributes, have direct effect on the selection procedure. The small number of attributes set-aside as "pertinent attributes" as necessitated by the particular application and/or the user (Prabhakarn *et al.*, 2006). The threshold values to these "relevant attributes" are assigned by obtaining information from the users and the group of experts. Henceforth, the selection process emphasizes on the pertinent attributes only. On the basis of the threshold values of the pertinent attributes, a shortlist of nanocomposite is obtained by scanning the data for pertinent attributes, one at a time, to eliminate the nanocomposite substitutes, which have one or more relevant attribute values that fall short of the minimum required (threshold) values. To facilitate that search procedure, an identification system has been made for the entire nanocomposite in the data.

Stage-2-Evaluation using TOPSIS method:
1. Decision matrix: The first step here to represent all the information available from the data about these satisfying solutions in the matrix form, known as decision matrix, D. Each row of the matrix is allocated to one candidate nanocomposite and each column to one attribute under consideration. Therefore, an element digit of the decision matrix D, provides the value of jth attribute in the row (non-normalized) form and units, for the ith nanocomposite. Thus, if the number of short-listed nanocomposite is "m" and the number of pertinent attributes is "n" the decision matrix is an m x n matrix. Therefore, an element d^{ij} of the decision matrix, D "represents the value of jth attribute in non-normalized form/units, corresponding to i^{th} alternative. Thus, if there are "m" Short-listed alternatives with n pertinent attributes the decision matrix is an m x n matrix.

2. Normalized specifications: After constructing decision matrix D, the normalized specification matrix, N is constructed from the decision matrix, D. Normalization is used to fetch the data within particular range and it provides the dimensionless magnitudes. The phenomenon is used to calculate the normalized specification matrix. The normalized specification matrix has the magnitudes of all the attributes of the nanocomposites on the common scale of 0-1. It is a type of value, which indicates the standing of that particular attribute magnitude when compared to the whole range of the magnitudes for all candidate nanocomposites.

An element n_{ij} of the normalized matrix N is obtained by:

$$n_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=j}^{m} d_{ij}^2}} \ldots\ldots\ldots\ldots 1$$

Where, d_{ij} is the element of the matrix D.

3. Relative importance matrix: In this step, the relative importance matrix 'R' of size n x n is formed to incorporate the relative importance of the attributes over other for a given application. An element r^{ij} of matrix $\underline{\underline{R}}$ represents the relative importance of the ith attribute over the jth attribute and is defined as:

r_{ij} = importance of ith attribute/importance of jth attribute……2

The relative importance of one attribute with respect to another for a given application can be obtained from the user or the group of experts specialized in a particular application. The information about the pair-wise comparison of attributes for a particular application is stored in this relative importance matrix $\underline{\underline{R}}$, with all its diagonal elements as unity.

4. Eigen value formulation and weight matrix: The information stored in the $\underline{\underline{R}}$ matrix on a pair-wise basis cannot be used directly. It must be modified into a form that gives the relative weights of all attributes taken together so that the sum of all the weight is equal to unity. Thus, Eigen value formulation is used to find weight vector matrix, $\underline{\underline{W}}$ and is expressed as:

RW = λW……3

Where W = {w1, w2, w3……..wn} T, and λ is the Eigen value.

Eq. (3) can be expressed as
$(R - \lambda I) = 0$……4

To avoid the trivial solution, we have
$(R - \lambda I) = 0$…… 5

The solution of Eq. (5) gives the set of n Eigen values (λ1, λ2… λn). The solution of Eq. (4) for the maximum Eigen value λmax gives the weight matrix, W and the expression is given as:

$(R - \lambda MAX\ I) = 0$…… 6

5. Weighted normalized decision matrix: In this step, the weighted normalized decision matrix, V is obtained by incorporating the information stored in the weight matrix, W into the normalized matrix, N. A true comparable value of each attribute is given by this weighted normalized matrix and is defined as:

V $=V_{ij}$, where $V_{ij} = W_{ij}$ x N_{ij}

where i = 1,2,….,m; j = 1,2,….,n.

6. Hypothetical best and worst solution: The hypothetical best solution (HBS) and hypothetical worst solution (HWS) are determined by choosing the maximum and minimum values of attributes from V matrix as:

HBS = A* $=V_{ij}\ max$, for benefit attributes (Larger the better type), or
$=V_{ij}\ min$, for cost attributes (Smaller the better type), and

HWS = A⁻ $=V_{ij}\ min$, for benefit attributes (Larger the better type), or
$=V_{ij}\ max$, for cost attributes (Smaller the better type)
Where i = 1, 2… m and j = 1,2,…,n.

Hence, A* = (V*1, V*2… V*n)
A⁻ = (V⁻1, V⁻2... V⁻ n)

7. Determination of separation measures and suitability index: The matrix V is used to obtain the positive and negative benchmark nanocomposite, where the both benchmark nanocomposites are hypothetical and supposed to have best and worst possible attribute magnitudes. TOPSIS method is based on the concept that the chosen option (optimum) have the shortest distance from the positive benchmark nanocomposite and be farthest from the negative benchmark nanocomposite (Bhangale *et al.,* 2004). The measure ensures that the top ranked nanocomposite is closest to positive benchmark nanocomposite and farthest from negative benchmark nanocomposite. Here, calculations are made on separation measures from positive and negative benchmark nanocomposite, respectively, as S_i^* and S_i.

The separation from the positive benchmark nanocomposite is given by:

$$s_i^* = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_1^*\right)^2} \quad (i = 1,2,\ldots.m) \ldots\ldots..7$$

And separation from the negative benchmark nanocomposite I is given by:

$$s_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_1^-\right)^2} \quad (i = 1,2,\ldots.m)$$

Then, the relative closeness to the positive benchmark nanocomposite, C*, which is a measure of the suitability of the nanocomposite for the chosen application on the basis of attributes considered, is calculated. A nanocomposite with the largest C* is preferable.

$C^* = S_i^-/(S_i^* + S_i^-)$……8

Ranking of the candidate nanocomposite in accordance with the decreasing values of indices C* indicating the most preferred and the least preferred feasible optional solutions is done.

8. Establishing an order of preference: Nanocomposite with highest value of C* will be given highest rank and so on. In this way the preference order for the available alternative nanocomposite is obtained by arranging them in decreasing order of their corresponding C* values**.**

9. Final selection: After the elimination search, the finite numbers of attributes which are sufficient to evaluate the system are shortlisted. On the basis of these attributes a number of designs are developed and after that the ranking nanocomposite is done mathematically by TOPSIS method.

Illustrative example of selection of nanocomposite: The above mentioned theory of the MADM-TOPSIS approach will be easily understood with the help of an illustrative example, given below. The "elimination search" easily found out manageable number of candidate nanocomposite and their pertinent attributes tabulated in Table 5. The minimum required attributes that are responsible for selection of these types of nanocomposite are:

i. Tensile strength
ii. Tensile modulus
iii. Elongation break (%)

The values of the properties of nanocomposite are taken from the scholarly article of Ke and Troeve (2006) in the field of Chemical engineering and Material science Dept. of China University of Petroleum (Beijing, China).

Table 5. Attributes for short listed candidate nanocomposite (Ke and Troeve, 2006).

Alternative nanocomposite	Tensile strength (Mpa)	Tensile modulus (Gpa)	Elongation break (%)
Nylon 6/Clay MMTs	97.2	1.87	7.3
Nylon 6/Clay SM	93.1	2.02	7.2
Nylon 6/Clay LiMMTs	89.5	1.65	100
Nylon 6/Clay' TALC	84.7	1.59	100

Selection of nanocomposite using MADM-TOPSIS approach:

Step 1: Formation of decision matrix, 'D': From Table 5, prepare a decision matrix. The matrix contains all the magnitudes of specifications. In this matrix, rows represent the candidate nanocomposite and the column represents the pertinent attributes. In this example, 4 candidate nanocomposite and 3 attributes are considered with their values are listed below.

$$D = \begin{pmatrix} 97.2 & 1.87 & 7.3 \\ 93.1 & 2.02 & 7.2 \\ 89.5 & 1.65 & 100 \\ 84.7 & 1.59 & 100 \end{pmatrix} \quad \ldots\ldots..9$$

Step 2: Construction of relative importance matrix A: After, identify the applications from different applications as per requirement, the relative importance of different attributes for different application is different. Now for selected application we define this as $a_{ij} = w_i/w_j$, where this ratio represents the relative importance of ith attribute with respect to the jth attribute corresponding to the particular application. Mostly this will be obtain by team of experts by preparing the questionnaires and send it to other team of experts of same area and prepare a database matrix by calculating the average values and put into the relative importance matrix. A group of experts will determine the relative importance of the attributes with respect to each other.

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 0.5 \\ 0.33 & 2 & 1 \end{pmatrix} \quad \ldots\ldots\ldots10$$

Step 3: Finding out the maximum Eigen value of the relative importance matrix A: Eigen value formulation provides to find out the weight vector as shown below:

$(A - \lambda I)\, W = 0$..... 11

Where, I is the identity matrix, and W is the weight vector.

This equation can also be written as $(A - \lambda\, I) = 0$ and $W = 0$, Where this gives a trivial solution having no meaning.

$$(A - \lambda\, I) = \begin{pmatrix} 1-\lambda & 1 & 3 \\ 1 & 1-\lambda & 0.5 \\ 0.33 & 2 & 1-\lambda \end{pmatrix} \quad \ldots.12$$

As $(A-\lambda I) = 0$, After solving the characteristics polynomial equation, by using the power method λmax is calculated. From this, λ=3.349 is the maximum value, as by taking the largest λ value the correct equation is obtained.

Therefore, λmax = 3.349. Now put this value in Equation

$$(A-\lambda max I) = \begin{pmatrix} -2.349 & 1 & 3 \\ 1 & -2.349 & 0.5 \\ 0.33 & 2 & -2.349 \end{pmatrix} \quad \ldots.13$$

Step 4: Calculating weights for each attribute using the Eigen vector associated with maximum Eigen value.

$(A- \lambda max\ I)\, W = 0$ ……. 14

$$(A-\lambda max I)W = \begin{pmatrix} -2.349 & 1 & 3 \\ 1 & -2.349 & 0.5 \\ 0.33 & 2 & - \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \quad \ldots.15$$

W1+W2+W3 = 1

W1=0.4647, W2=0.2571, W3=0.2783……..16

Step 5: Calculating the normalized data matrix _N_ using Eq. (1):

$$n_{ij} = d_{ij} / \sqrt{\sum_{i=j}^{m} d_{ij}^2} \ldots\ldots.17$$

$$N = \begin{pmatrix} 0.5327 & 0.5221 & 0.0514 \\ 0.5101 & 0.5639 & 0.0507 \\ 0.4905 & 0.4606 & 0.7052 \\ 0.4441 & 0.4439 & 0.7052 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots18$$

The normalization helps to provide the dimensionless elements of the matrix.

Step 6: Calculating the weighted normalized specification matrix:

$$V = \begin{pmatrix} 0.2475 & 0.1342 & 0.0143 \\ 0.2370 & 0.1449 & 0.0141 \\ 0.2279 & 0.1184 & 0.1962 \\ 0.2156 & 0.1141 & 0.1962 \end{pmatrix} \quad \ldots\ldots\ldots 19$$

The matrix will be able to provide good basis for comparison with each other and with the benchmark nanocomposites.

Step 7: Determine positive ideal and negative ideal solution using the relation: The weighted normalized attributes for the positive benchmark nanocomposite is calculated by taking the largest value from all the columns with respect to all the pertinent attributes in correspondence to their candidate nanocomposite. Theoretically best solution of nanocomposite:

$V^* = 0.2475, 0.1449, 0.1962$

Similarly, the weighted normalized attributes for the negative benchmark nanocomposite is calculated by taking the smallest value from all the columns with respect to all the pertinent attributes in correspondence to their candidate nanocomposite. Theoretically worst solution for nanocomposite:

$V^- = 0.2156, 0.1141, 0.0141$

Values of separation from the positive benchmark nanocompositess* and values of separation from the negative benchmark nanocompositess$^-$ are given below:

$S_1^* = 0.1822$ $S_1^- = 0.0377$
$S_2^* = 0.1824$ $S_2^- = 0.0375$
$S_3^* = 0.0329$ $S_3^- = 0.1825$
$S_4^* = 0.0443$ $S_4^- = 0.1821$

Step 8: Relative closeness to ideal solution:
$c_1^* = 0.1714$, $c_2^* = 0.1709$, $c_3^* = 0.3568$, $c_4^* = 0.8043$

Evaluation and ranking of the candidate nanocomposite using TOPSIS methods is shown in Table 6.

Graphical representation: TOPSIS method is suitable to carry out attribute-based evaluation using computer. To develop basic understanding of multi-attribute decision-making approach, graphical procedures are also proposed for evaluation of candidates.

Table 6. Evaluation and ranking of the candidate nanocomposite using TOPSIS method.

Candidate TOPSIS nanocomposite	TOPSIS closeness to positive benchmark C*	Rank based on C*
Nylon 6/Clay MMTs	0.1714	3
Nylon 6/Clay SM	0.1709	4
Nylon 6/Clay LiMMTs	0.3568	2
Nylon 6/Clay' TALC	0.8043	1

The graphical representation methods like line graph and spider diagram are used for this purpose. These graphical representations help in enhancing the insight and better understanding of the available subsystems and systems (Prabhakarn *et al.,* 2006).

Line graph representation: From the above analysis, we developed decision matrix D, normalized and weighted normalized data matrices N and V, respectively, having information of various pertinent attributes of various types of candidates chosen for study. These matrices can be represented graphically using line graph by plotting the magnitude of the attributes on the vertical axis and the attributes on the horizontal axis. It may be noted that the attributes, of which minimum values are preferred such as accuracy, repeatability, cost, etc., we use the reciprocals of the magnitudes to plot so that it will bring consistency where all the attributes are to be maximized to reach the best possible solution. If we plot the values for different candidates, we can obtain the line graph for them. These graphs will be distinct for all the types of candidate and can be used as comparison basis. The area under the curve can be used to quantification purpose and to compare the different kinds of subsystems with each other and benchmark the best selection of candidate for the given application, to be defined later. The ordinates of different attributes in the line graph represent either d_{ij}, n_{ij} or p_{ij} while, abscissa of different attributes are at unit distances apart. These line graphs can be plotted for decision, normalized and weighted normalized data matrices of all the candidates as well as the benchmark candidates. The Trapezium areas "Ai" under the curve can be obtained as follows. Let the width between the two parameters on horizontal axis as unity and d_{ij}, n_{ij}, and v_{ij} are the elements of D, N, and V matrices. Area under the line graph of specification of ith nanocomposite found out as:

$$AD_i^L = \frac{\left[d_{i.1} + 2\left(d_{i.2} + \cdots \ldots\ldots + d_{i.n-1} \right) + d_{i.n} \right]}{2} \ldots\ldots..20$$

Similarly, area under the graph of normalized and weighted normalized specifications of the Ith nanocomposite, i.e. AN_i^L and AN_i^L using their respective elements. Figure 4 shows line graph diagram for three attributes based on database matrix for four candidate reinforcement systems as given in illustrative example.

Fig. 4. Line graph plot for the database matrix and the area under the curve.

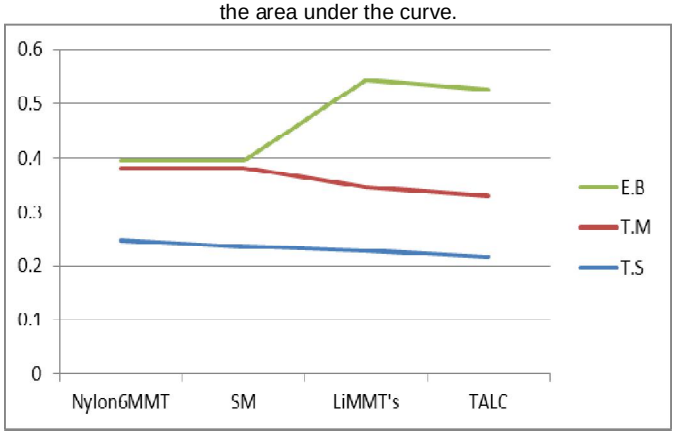
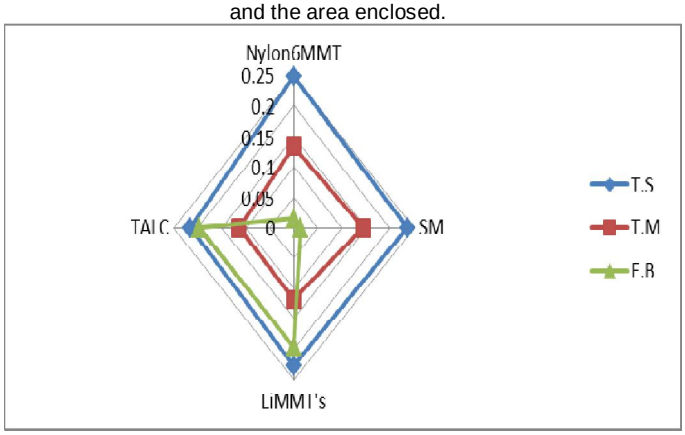


Fig. 5. Spider diagram polygon for the database matrix and the area enclosed.



Spider diagram representation: In this method, the attributes have been considered to be forming the spider diagram. So the angle θ between the attribute axes passing through a common point O can be calculated as θ=2θ/n, where n is number of attributes under consideration. The attributes data d_{ij}, normalized r_{ij} and weighted normalized p_{ij} data matrices magnitudes are plotted to obtain the spider diagram, also known as polar\diagram. Here, the area enclosed by the polygon formed by joining attribute magnitudes on corresponding attribute axes on the spider diagram is the indication of the candidate capabilities. All the magnitudes of candidates are boiled down to this single index. This area enclosed by the polygon of the ith candidate can be calculated as follows. In the spider diagram, θ=2π/n, where n is the number of attributes. Let d_{ij} represents the value of jth attribute in the ith candidate along θ_i. Let r_{ij} represents the normalized value of the jth attribute in the ith candidate along θ_i. Let p_{ij} represents the weighted normalized value of the jth attribute in the ith candidate along θ_i.

$$D_s^i = \sin\ \theta/2\ \sum_j^n d_{ij}\ \ d_{j+1} \ldots\ldots\ldots (21)$$

Similarly, areas for normalized and weighted normalized data matrices of various candidates as areas enclosed by polygons, i.e., N_i^s and V_i^s, respectively, are calculated. Figure 5 shows spider diagram for four attributes of five candidate reinforcement system as given in illustrative example.

Graphical method based ranking: The element values of weighted normalized specification matrix are used for the line graph plotting. Subsequently, COS (Coefficient of similarity) can be calculated from graphs. The calculated COS (Coefficient of similarity) is tabulated. Suppose the area under the line graph for weighted normalized specifications of first candidate nanocomposite and for benchmark nanocomposite are:

AV_1^L=0.5302, AV_{+B}^L=0.7335

The coefficient of similarity based on the weighted normalized specification of the first candidate nanocomposite is:

$$COS_1^{VL} = \frac{AV_1^L}{AV_{+B}^L} = 0.7228 \ldots\ldots\ldots (22)$$

Similarly, closeness of the candidate nanocomposite with the positive benchmark nanocomposite obtained from TOPSIS and the graphical methods are shown in Table 7. Thus, the nanocomposites are ranked in order of preference based on the attributes selected.

Results and discussion

Outcome of TOPSIS approach: Table 5 shows selected nanocomposites and attributes required by manufacturer. After applying TOPSIS approach to these attributes, the values nanocomposites close to ideal solution V^* are calculated and are denoted by c^*. Then, these nanocomposites are ranked according to the values of c^*, ranking of candidate nanocomposites is given in Table 6, Where, rank 1 means highly suitable and rank 4 means least suitable.

Outcome of graphical representation: In graphical representation, the graphs were plotted by taking values of elements of weighted normalized specification matrix. After plotting the graph, COS (Coefficient of similarity) is calculated for all candidate nanocomposites and ranking is done on the base closeness to positive benchmark (Table 7).

Table 7. Evaluation and ranking of the candidate nanocomposite using line graphical method.

Candidate TOPSIS nanocomposite COS^{VL}	COS*	Rank based on C*
Nylon 6/Clay MMTs	0.7228	4
Nylon 6/Clay SM	0.7374	3
Nylon 6/Clay LiMMTs	0.9010	1
Nylon 6/Clay' TALC	0.8725	2

Final selection of best optimum result: The ranking can be done accordance with the decreasing values of c_i^* and cos^{vl}. The values of c_i^* and cos^{vl} indicates the most preferred optional solution. The ranking is also obtained through graphical method. These two methods give the optimum solution as a nylon/clay Talc and nylon/clay LiMMT's as a best solution, for the given attributes. Table 8 shows the most suitable nanocomposites collectively ranked by both the approaches.

Conclusion

MADM-TOPSIS is an effective method, which is suitable for solving multiple attribute group decision making problems under a fuzzy environment where, the information available is subjective and imprecise. The proposed method enables a group of decision makers to incorporate and aggregate subjective opinions. The basic principle of the TOPSIS method is that the chosen alternative should have the farthest distance from the negative ideal solution and shortest distance from the positive ideal solution. In this chapter a new method is proposed to balance the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The MADM-TOPSIS can be employed as an alternative tool for situations where both qualitative and quantitative data has to be synthesized. By using the developed model the designers and manufacturer can choose the best nanocomposite based on the requirements of attributes including tensile strength, tensile modulus and elongation break.

Table 8. Best optimum result.

Candidate nanocomposite	Rank (best optimum result)
Nylon 6/Clay LiMMTs	1
Nylon 6/Clay' TALC	2

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